Probing signals of the littlest Higgs model via the WW fusion processes at the high energy e^+e^- collider

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Abstract

In the framework of the littlest Higgs(LH) model, we consider the processes $e^+e^- \rightarrow \nu\bar{\nu}H^0$ and $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$, and calculate the contributions of new particles to the cross sections of these processes in the future high energy e^+e^- collider(ILC) with $\sqrt{S} = 1TeV$. We find that, with reasonable values of the free parameters, the deviations of the cross sections for the processes $e^+e^- \rightarrow \nu\bar{\nu}H^0$ from its SM value might be comparable to the future ILC measurement precision. The contributions of the light Higgs boson H^0 to the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ are significant large in all of the parameter space preferred by the electroweak precision data, which might be detected in the future ILC experiments. However, the contributions of the new gauge bosons B_H and Z_H to this process are very small.

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I. Introduction

Little Higgs models[1,2,3] employ an extended set of global and gauge symmetries in order to avoid the one-loop quadratic divergences and thus provide a new method to solve the hierarchy between the TeV scale of possible new physics and the electroweak scale $\nu = 246 GeV = (\sqrt{2}G_F)^{-1/2}$. The key feature of this type of models is that the Higgs boson is a pseudo-Goldstone boson of a global symmetry which is spontaneously broken at some higher scale f and thus is naturally light. Electroweak symmetry breaking($EWSB$) is induced by a Coleman-Weinberg potential, which is generated by integrating out the heavy degrees of freedom. This type of models can be regarded as one of the important candidates of the new physics beyond the standard model(SM).

The next generation of high energy e^+e^- linear colliders($ILC's$) are expected to operate at the center-of -mass(c.m.) energy $\sqrt{S} = 300 GeV - -1.5 TeV$, which are required to complement the probe of the new particles with detailed measurement[4]. They will offer an excellent opportunity to study the dynamics of the new physics with uniquely high precision. The main production mechanism of the neutral Higgs boson in these collider experiments are the Higgs-strahlung process $e^+e^- \to ZH^0$ and the WW fusion process $e^+e^- \to W^*W^*\nu\overline{\nu} \to \nu\overline{\nu}H^0$[5]. The cross section for the Higgs-strahlung process scales as $1/S$ and dominates at low energies, while the cross section for the WW fusion process rises $\log(S/m_H^2)$ and dominates at high energies. It has been shown that, for $\sqrt{S} \geq 500 GeV$, the WW fusion contributions dominate the total cross section for the Higgs production processes[6]. The ZZ fusion process $e^+e^- \to Z^*Z^*e^+e^- \to e^+e^-H^0$ can also contribute to the Higgs boson production. However, the cross section is suppressed by an order of magnitude compared to that for the WW fusion process, due to the ratio of the $W^\pm e\nu_e$ coupling to the Zee coupling, $4c_W^2 = 3$.

In Ref.[7], we have calculated the cross section of the Higgs-strahlung process $e^+e^- \to ZH^0$ in the context of the littlest Higgs(LH) model[1]. We find that, in most of the parameter space, the deviation of the total cross section from its SM value is larger than 5%, which may be detected at the future ILC experiment with $\sqrt{S} = 500 GeV$. In this paper, we will consider the WW fusion process $e^+e^- \to W^*W^*\nu\overline{\nu} \to \nu\overline{\nu}H^0$ and see

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whether the light Higgs boson predicted by the LH model can be detected via this process at the future ILC experiment with $\sqrt{S} = 1TeV$.

It is well known that vector boson scattering processes can be used to probe kinds of $EWSB$ mechanism at TeV energies[8]. The WW fusion process $W^+W^- \to t\bar{t}$ could be used to probe how the Higgs sector couples to fermions. Although QCD backgrounds make this process very difficult to observe at the hadron colliders, it has been shown[9] that the signals of the SM Higgs sector could be established with good statistical significance at the ILC with $\sqrt{S} = 1.5TeV$. In this paper, we will study the WW fusion process $W^+W^- \to t\bar{t}$ at the future ILC with $\sqrt{S} = 1TeV$. In the context of the LH model, we calculate the contributions of the light Higgs boson H^0 to this process and further calculate the cross section for the process $e^+e^- \to W^*W^*\nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ using the effective W-boson approximation (EWA)[10]. We find that the cross section of this process is very sensitive to the free parameters of the LH model and the possible signals of the little Higgs boson H^0 should be detected at the future ILC experiments with $\sqrt{S} = 1TeV$.

The LH model predicts the existence of the heavy gauge bosons, such as Z_H and B_H. We further study the contributions of these new gauge bosons to the WW fusion process $e^+e^- \to W^*W^*\nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ in this paper. We find that the contributions of gauge boson Z_H exchange and B_H exchange to this process are very small in all of the parameter space preferred by the electroweak precision data, which can not be detected in the future ILC experiments.

In the next section, we give the couplings and masses of the new particles predicted by the LH model, which are related to our calculation. In Sec.III we calculate the single production cross-section of the light Higgs boson H^0 via the WW fusion process and compare our numerical result with that given in the SM. The contributions of the little Higgs boson H^0 to the process $W^+W^- \to t\bar{t}$ are studied in Sec.IV. Using the EWA method, we further calculate the cross section for the process $e^+e^- \to W^*W^*\nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ generated by H^0 exchange in this section. The possible contributions of the heavy gauge bosons to the process $e^+e^- \to W^*W^*\nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ are studied in Sec.V. Our conclusions and discussions are given in Sec.VI.

II. The relevant coupling forms

The LH model[1] is one of the simplest and phenomenologically viable models, which realizes the little Higgs idea. It consists of a non-linear σ model with a global $SU(5)$ symmetry and a locally gauged symmetry $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$. The global $SU(5)$ symmetry is broken down to its subgroup $SO(5)$ by a vacuum condensate $f \sim \Lambda s/4\pi \sim TeV$, which results in fourteen massless Goldstone bosons. Four of these particles are eaten by the SM gauge bosons, so that the locally gauged symmetry $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ is broken down to its diagonal subgroup $SU(2) \times U(1)$, identified as the SM electroweak gauge group. The remaining ten Goldstone bosons transform under the SM gauge group as a doublet H and a triplet Φ. The doublet H becomes the SM Higgs doublet, while the triplet Φ is an addition to the SM particle contents. This breaking scenario also gives rise to the new gauge bosons $W_H^{\pm}, B_H, Z_H$.

In the LH model, the light Higgs boson acquires the mass squared parameter at two-loop as well as at one-loop from the Coleman-Weinberg potential. Its mass is protected from the one-loop quadratic divergence by a few new particles with the same statistics as the corresponding SM particles. The new heavy gauge bosons $W_H^{\pm}, B_H, Z_H$ cancel the one-loop quadratic divergence generated by the SM gauge boson W and Z loops. New heavy scalar Φ cancels that generated by the Higgs self-interaction. A new vector-like top quark T is also needed to cancel the divergences from the top quark Yukawa interactions. Furthermore, these new particles might produce characteristic signatures at the present and future collider experiments[7,11,12,13]. Certainly, these new particles can generate significant corrections to some observables and thus the precision measurement data can give severe constraints on this kind of models[11,14,15,16].

In the LH model, the coupling expressions of the Higgs boson H^0, which are related to our calculation, can be written as[11]:

$$g^{H^0 W_\mu^+ W_\nu^-} = \frac{ie^2 \nu g_{\mu\nu}}{2s_W^2}[1 - \frac{\nu^2}{3f^2} + \frac{1}{2}(c^2 - s^2)^2 \frac{\nu^2}{f^2} - 12\frac{\nu'}{\nu}], \tag{1}$$

$$g^{H^0 W_{H\mu}^+ W_{H\nu}^-} = -\frac{ie^2 \nu}{2s_W^2} g_{\mu\nu}, \qquad g^{H^0 W_\mu^+ W_{H\nu}^-} = -\frac{ie^2 \nu g_{\mu\nu}}{2s_W^2} \frac{(c^2 - s^2)}{2sc}, \tag{2}$$

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$$g^{H^0 t\bar{t}} = -\frac{im_t}{\nu}[1 - 4(\frac{\nu'}{\nu})^2 + 2\frac{\nu'}{f} - \frac{2}{3}(\frac{\nu}{f})^2 + \frac{\nu^2}{f^2}x_L(1 + x_L)]. \tag{3}$$

Where $s_W = \sin\theta_W$, θ_W is the Weinberg angle, ν' is vacuum expectation value(VEV) of the triplet scalar Φ. $c(s = \sqrt{1 - c^2})$ is the mixing parameter between $SU(2)_1$ and $SU(2)_2$ gauge bosons and the mixing parameter $c'(s' = \sqrt{1 - c'^2})$ comes from the mixing between $U(1)_1$ and $U(1)_2$ gauge bosons. Using these mixing parameters, we can represent the SM gauge coupling constants as $g = g_1 s = g_2 c$ and $g' = g_1' s' = g_2' c'$. The mixing parameter between the SM top quark t and the vector-like top quark T is defined as $x_L = \lambda_1^2/(\lambda_1^2 + \lambda_2^2)$, in which λ_1 and λ_2 are the Yukawa coupling parameters.

Taking account of the gauge invariance of the Yukawa couplings and the $U(1)$ anomaly cancellation, the relevant couplings of the gauge bosons W, $W_H^{\pm}$, B_H, and Z_H to ordinary particles can be written as in the LH model:

$$g_L^{W\nu e} = \frac{ie}{\sqrt{2}s_W}[1 - \frac{\nu^2}{2f^2}c^2(c^2 - s^2)], \quad g_R^{W\nu e} = 0; \tag{4}$$

$$g_L^{W_H \nu e} = -\frac{ie}{\sqrt{2}s_W}\frac{c}{s}, \quad g_R^{W_H \nu e} = 0; \tag{5}$$

$$g_L^{Wtb} = \frac{ie}{\sqrt{2}s_W}[1 - \frac{\nu^2}{2f^2}(x_L^2 + c^2(c^2 - s^2))], \quad g_R^{Wtb} = 0; \tag{6}$$

$$g^{B_H W_\mu^+ W_\nu^-} = -\frac{ec_W}{s_W^2}[\frac{\nu^2}{f^2}\frac{5}{2}s'c'(c'^2 - s'^2)], \quad g^{Z_H W_\mu^+ W_\nu^-} = \frac{e}{2s_W}[\frac{\nu^2}{f^2}sc(c^2 - s^2)]; \tag{7}$$

$$g_L^{B_H t\bar{t}} = \frac{e}{6c_W s'c'}(\frac{2}{5} - c'^2), \quad g_R^{B_H t\bar{t}} = \frac{2e}{3c_W s'c'}[(\frac{2}{5} - c'^2) - \frac{3}{20}x_L]; \tag{8}$$

$$g_L^{Z_H tt} = \frac{ec}{2s_W s}, \quad g_R^{Z_H tt} = 0. \tag{9}$$

To obtain our numerical results, we write the masses of the relevant particles as:

$$M_W^2 = m_W^2[1 - \frac{\nu^2}{f^2}(\frac{1}{6} + \frac{1}{4}(c^2 - s^2)^2) + 4(\frac{\nu'^2}{\nu^2})], \tag{10}$$

$$M_{W_H}^2 \approx m_W^2(\frac{f^2}{s^2c^2\nu^2} - 1), \quad M_{B_H}^2 \approx \frac{m_W^2 s_W^2}{c_W^2}[\frac{f^2}{5s'^2c'^2\nu^2} - 1], \tag{11}$$

$$M_{Z_H}^2 \approx M_{W_H}^2, \tag{12}$$

where $m_W = g\nu/2$ is the mass of the SM gauge boson W. From above equations, we can see that, at the order of ν^2/f^2, the B_H mass M_{B_H} and the Z_H mass M_{Z_H} mainly depend on the free parameters (f, c') and (f, c), respectively. In general, the heavy

photon B_H is substantially lighter than the gauge boson Z_H. Considering the constraints of the electroweak precision data on the free parameters f, c, and c', the value of the ratio $M_{B_H}^2/M_{Z_H}^2$ can be further reduced.

In the following calculation, we will take the mass of the light Higgs boson $m_H = 115GeV$. In this case, the possible decay modes of H^0 are $b\bar{b}$, $c\bar{c}$, $l\bar{l}$ [$l = \tau, \mu$ or e], gg and $\gamma\gamma$. However, the total decay width Γ_H is dominated by the decay channel $H^0 \rightarrow b\bar{b}$. In the LH model, Γ_H is modified from that in the SM by the order of ν^2/f^2 and has been studied in Ref.[13].

Considering the electroweak precision data constraints, the B_H mass M_{B_H} is not too heavy and can be allowed to be in the range of a few hundred GeV[14]. For the decay channels $B_H \rightarrow \bar{t}t$ and $B_H \rightarrow ZH$, we can not neglect the final state masses. The electroweak precision data constrain the Z_H mass M_{Z_H} to be no smaller than about $1TeV$. Thus, for all of the Z_H decay channels, we can neglect the final state masses. The total decay widths Γ_{Z_H} and Γ_{B_H} of the gauge bosons Z_H and B_H have been discussed in Refs.[13,14]. It is easily to know that Γ_{B_H} is sensitive to the free parameters f and c', while Γ_{Z_H} is sensitive to the free parameters f and c.

Global fits to the electroweak precision data produce rather severe constraints on the parameter space of the LH model[14, 15]. However, if the SM fermions are charged under $U(1)_1 \times U(1)_2$, the constraints become relaxed. The scale parameter $f = 1 \sim 2TeV$ is allowed for the mixing parameters c, c', and x_L in the ranges of $0 \sim 0.5$, $0.62 \sim 0.73$, and $0.3 \sim 0.6$, respectively[16]. Taking into account the constraints on the free parameters f, c, c' and x_L, we will give our numerical results in the following sections.

III. The WW fusion process $e^+e^- \rightarrow \nu\bar{\nu}H^0$ in the LH model

A future ILC will measure the production cross section of a light Higgs boson via WW fusion with percent-level precision[4]. Furthermore, in the ILC experiments with $\sqrt{S} \geq 500GeV$, the WW fusion process $e^+e^- \rightarrow \nu\bar{\nu}H^0$ dominates single production of the Higgs boson H^0[6]. Thus, it is very interesting to study this process in the popular specific models beyond the SM.

In the SM, the production cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}H^0$ can be generally

written as[5]:

$$\sigma^{SM} = \frac{G_F^3 m_W^4}{4\sqrt{2}\pi^3} \int_{x_H}^1 dx \int_x^1 \frac{dy F(x,y)}{[1+(y-x)/x_W]^2} \tag{13}$$

with

$$F(x,y) = (\frac{2x}{y} - \frac{1+3x}{y^2} + \frac{2+x}{y} - 1)[\frac{z}{1+z} - \ln(1+z)] + \frac{xz^2(1-y)}{y^3(1+z)}, \tag{14}$$

where $x_H = m_H^2/S$, $x_W = m_W^2/S$, and $z = y(x-x_H)/(xx_W)$.

Compared with the WW fusion process $e^+e^- \to \nu\bar{\nu}H^0$ in the SM, this process in the LH model receives additional contributions from the heavy gauge bosons $W_H^\pm$, proceed through the Feynman diagrams depicted in Fig.1. Furthermore, the modification of the relations among the SM parameters and the precision electroweak input parameters, and the correction terms to the SM $We\nu_e$ coupling can also produce corrections to this process.
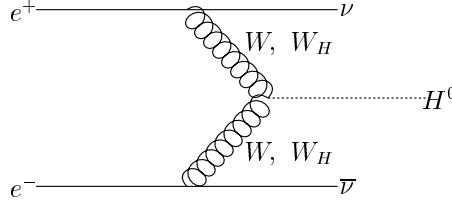

Figure 1: Feynman diagrams for the WW fusion process $e^+e^- \to \nu\bar{\nu}H^0$ in the LH model.

In the LH model, the relation among the Fermi coupling constant G_F, the gauge boson W mass m_W and the fine structure constant α can be written as[16]:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2m_W^2 s_W^2}[1 - c^2(c^2-s^2)\frac{\nu^2}{f^2} + 2c^4\frac{\nu^2}{f^2} - \frac{5}{4}(c'^2-s'^2)\frac{\nu^2}{f^2}]. \tag{15}$$

So we have

$$\frac{e^2}{s_W^2} = \frac{4\sqrt{2}G_F m_W^2}{[1 - c^2(c^2-s^2)\frac{\nu^2}{f^2} + 2c^4\frac{\nu^2}{f^2} - \frac{5}{4}(c'^2-s'^2)\frac{\nu^2}{f^2}]}. \tag{16}$$

In the following numerical estimation, we will take $G_F = 1.16637 \times 10^{-5} GeV^{-2}$, $m_Z = 91.18 GeV$ and $m_W = 80.45 GeV$[17] as input parameters and use them to represent the

other SM parameters. The $c.m.$ energy $\sqrt{S}$ of the future ILC experiments is assumed as $\sqrt{S} = 1TeV$.
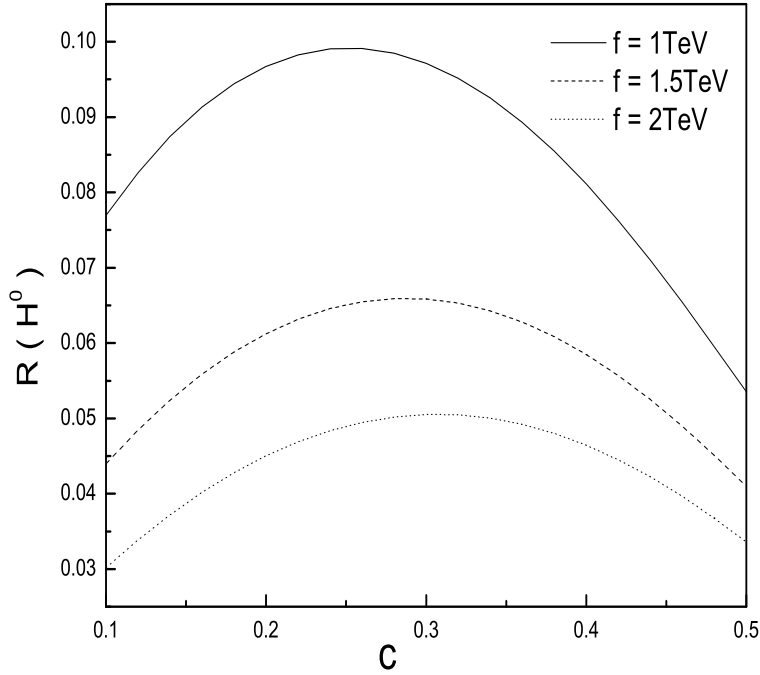

Figure 2: The parameter $R(H^0)$ as a function of the mixing parameter c for $\nu'/\nu = \nu/5f$ and three values of the scale parameter f.

Except for the SM input parameters, there are three free parameters in the expression of the relative correction parameter $R(H^0) = \Delta\sigma/\sigma^{SM}$ with $\Delta\sigma = \sigma^{LH} - \sigma^{SM}$: the mixing parameter c, the scale parameter f, and the triplet scalar VEV ν'. In order to obtain the correct $EWSB$ vacuum and avoid giving a TeV-scale VEV to the scalar triplet Φ, we should have that the value of ν'/ν is smaller than $\nu/4f$[1, 11]. In Fig.2, we plot the relative correction parameter $R(H^0)$ as a function of the mixing parameter c for $\nu'/\nu = \nu/5f$ and three values of the scale parameter f. From Fig.2, we can see that the value of $R(H^0)$ decreases as f increasing, which is consistent with the conclusions for the corrections of the LH model to other observables. If we assume $f = 1TeV$, the value of the relative correction parameter $R(H^0)$ is larger than 5.4% in all of the parameter space preferred by the electroweak precision data. For $f \geq 2TeV$, $R(H^0)$ is smaller than 5% in most of

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the parameter space of the LH model.

To see the effects of the varying triplet scalar VEV ν' on the relative correction parameter $R(H^0)$, we take $f = 1TeV$, which means $\nu'/\nu \leq \nu/4f = 0.061$, and plot $R(H^0)$ as a function of ν'/ν in Fig.3 for three values of the mixing parameter c. One can see from Fig.3 that $R(H^0)$ is not sensitive to the ratio ν'/ν, compared with the mixing parameter c. For $f = 1TeV$ and $\nu'/\nu \leq 0.06$, the value of $R(H^0)$ is larger than 4% and 6% for the mixing parameter $c = 0.1$ and 0.3, respectively.
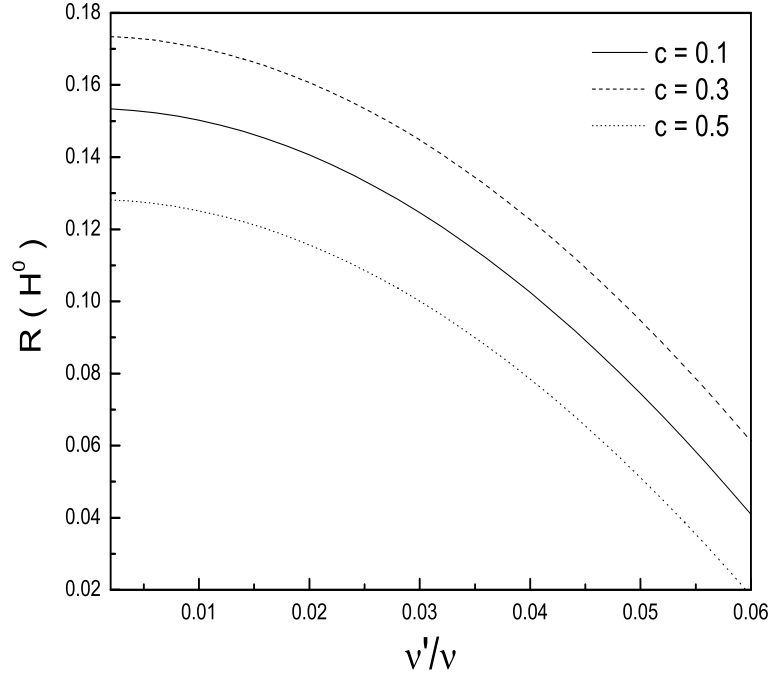

Figure 3: The parameter $R(H^0)$ as a function of ν'/ν for $f = 1TeV$ and three values of the mixing parameter c.

In general, the LH model can produce corrections to single production of the light Higgs boson H^0 via the WW process $e^+e^- \rightarrow \nu\bar{\nu}H^0$ at the future ILC experiments. Our results show that the correction effects on the production cross section can be significant large in all of the parameter space of the LH model. Even if we take account of the constraints of the electroweak precision data on the free parameters of the LH model,

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the value of the relative correction parameter $R(H^0)$ is generally larger than 5%. A future ILC will measure the production cross section of a light Higgs boson from Higgs-strahlung or WW fusion process with percent-level precision, as well as the important branching fractions with few-percent precision[4,18]. Thus, correction effects of the LH model on the WW fusion process $e^+e^- \rightarrow \nu\bar{\nu}H^0$ might be comparable to the future ILC measurement precision.

IV. The Higgs boson H^0 and the process $W^+W^- \rightarrow t\bar{t}$ in the LH model

The production cross section of the process $W^+W^- \rightarrow t\bar{t}$ generated by the Higgs boson H^0 is sensitive to the terms proportional to the coupling parameters $(g^{H^0 f\bar{f}})^2$ and $g^{H^0 f\bar{f}}$ of the Higgs boson H^0 to fermions, which come from pure Higgs contributions and the interference with non-Higgs contributions, respectively. Thus, the process $W^+W^- \rightarrow t\bar{t}$ could be used to probe how the Higgs sector couples to fermions. Although QCD backgrounds make this process very difficult to observe at the hadron colliders, the signals of the Higgs sector could be established with good statistical significance at the high energy ILC experiments[8,9,19]. In this section, we consider the contributions of the Higgs boson H^0 to this process in the context of the LH model and calculate the relative deviations from the SM prediction.

The subprocess $W^+W^- \rightarrow t\bar{t}$ can be effectively realized via gauge boson W radiation from initial fermion lines:

$$e^+e^- \rightarrow \nu\overline{\nu}W^*W^* \rightarrow \nu\overline{\nu}t\bar{t}, \tag{17}$$

which was first calculated in Ref.[20] by using the EWA method[10]. In the approach of an effective Lagrangian, Ref.[21] has extensively studied this process. Effects of the models of strong interaction $EWSB$ on the subprocess $W^+W^- \rightarrow t\bar{t}$ were discussed in Ref.[8].

For large $\sqrt{\hat{S}}$, which is the c.m. energy of the subprocess $W^+W^- \rightarrow t\bar{t}$ in the ILC with the c.m. energy $\sqrt{S} = 1 TeV$, the longitudinal polarization vector of gauge bosons $W^\pm$ can be approximately expressed by $\varepsilon_0^\mu(k) \approx k^\mu/m_W + O(m_W/\sqrt{\hat{S}})$. The term k^μ/m_W produces the leading contributions to the cross section $\hat{\sigma}(\hat{S})$ for the subprocess $W^+W^- \rightarrow t\bar{t}$, which are proportional to $(m_t/m_W)^4$, while the sub-leading contributions generated by the term

$O(m_W/\sqrt{\hat{S}})$ are suppressed by a factor $m_t^2/\hat{S}$. Thus, the production cross section $\hat{\sigma}(\hat{S})$ for the subprocess $W^+W^- \to t\bar{t}$ is well approximated by taking only the longitudinal polarized W's at the parton level reaction and assuming $\hat{S} \geq m_W^2$[10,20,21,22]. However, in this paper, we want to calculate the contributions of the Higgs boson H^0 to the cross section for the subprocess $W^+W^- \to t\bar{t}$ in the LH model and compare our numerical result with that in the SM. Thus, we will include all polarizations for the gauge bosons $W^\pm$ in our calculation of the production cross section $\hat{\sigma}(\hat{S})$.

In the LH model, the production cross section $\hat{\sigma}(\hat{S})$ for the subprocess $W^+_{\lambda_+} W^-_{\lambda_-} \to t\bar{t}$ generated by the Higgs boson H^0 can be written as:

$$
\begin{aligned}
\hat{\sigma}(\hat{S}) &= \frac{3\pi\alpha^2 A^2}{2s_W^4} \cdot m_t^2 X_H^2 \beta_t^3 \mid \varepsilon_{\lambda_+}^{W^+} \cdot \varepsilon_{\lambda_-}^{W^-} \mid^2 \\
&+ \frac{3\pi\alpha^2 AB^2}{8s_W^4} \cdot \frac{m_t^2}{\hat{S}} X_H \beta_t (1 - \beta_t^2) \\
&\cdot \{\mid \varepsilon_\pm^{W^+} \cdot \varepsilon_\pm^{W^-} \mid^2 [-1 + \frac{1+\beta_t^2}{2\beta_t^2} L] \\
&+ 4 \mid \varepsilon_0^{W^+} \cdot \varepsilon_0^{W^-} \mid^2 \cdot [-\frac{1+\beta_t^2}{1-\beta_t^2} + \frac{(1-\beta_t^2)}{2\beta_t^2} L]\}
\end{aligned}
\tag{18}
$$

with

$$
A = [1 - \frac{\nu^2}{3f^2} + \frac{\nu^2}{2f^2}(c^2 - s^2) - 12(\frac{\nu'}{\nu})^2] \cdot [1 - 4(\frac{\nu'}{\nu})^2 + 2\frac{\nu'}{f} - \frac{2}{3}\frac{\nu^2}{f^2} + \frac{\nu^2}{f^2} x_L(1 + x_L)], \tag{19}
$$

$$
B = 1 - \frac{\nu^2}{2f^2}[c^2(c^2 - s^2) + x_L^2], \qquad L = \ln(\frac{1+\beta_t}{1-\beta_t}), \tag{20}
$$

$$
\beta_t = \sqrt{1 - \frac{4m_t^2}{\hat{S}}}, \qquad X_H = \frac{\hat{S} - m_H^2}{(\hat{S} - m_H^2)^2 + m_H^2 \Gamma_H^2}. \tag{21}
$$

The second term of $Eq.(18)$ comes from the interference effects of the s-channel H^0 exchange with the t-channel b quark exchange. Due to the orthogonality properties of the polarizations vectors $\varepsilon_{\lambda_\pm}^{W^\pm}$ of the gauge bosons $W^\pm$, there is no interference between the transverse and the longitudinal polarizations. So we have

$$
\mid \varepsilon_\pm^{W^+} \cdot \varepsilon_\mp^{W^-} \mid^2 = 0, \quad \mid \varepsilon_\pm^{W^+} \cdot \varepsilon_\pm^{W^-} \mid^2 = 1, \quad \mid \varepsilon_0^{W^+} \cdot \varepsilon_0^{W^-} \mid^2 = \frac{(1+\beta_W^2)^2}{(1-\beta_W^2)^2} \tag{22}
$$

with $\beta_W = \sqrt{1 - 4m_W^2/\hat{S}}$.

In general, the cross section $\sigma(S)$ for the process $e^+ e^- \to \nu \bar{\nu} W^* W^* \to \nu \bar{\nu} t \bar{t}$ can be obtained by folding the cross section $\hat{\sigma}(\hat{S})$ for the subprocess $W^+_{\lambda_+} W^-_{\lambda_-} \to t \bar{t}$ with the $W^\pm$ distribution functions $f^{W^\pm}_{\lambda_\pm}$ with helicities $\lambda_\pm$:

$$\sigma(S) = \Sigma_{\lambda_+,\lambda_-} \int_{2m_t/\sqrt{s}}^{1} 2x dx \int_{x^2}^{1} \frac{dx_+}{x_+} f^{W^+}_{\lambda_+}(x_+) f^{W^-}_{\lambda_-}(\frac{x^2}{x_+}) \hat{\sigma}(\hat{S}), \qquad (23)$$

where $x^2 = \hat{S}/S$, the helicities $\lambda_\pm$ of the gauge bosons $W^\pm$ each run over 1, 0, -1. In our calculations, we use the full distributions given by Refs.[10,20] for $f^{W^\pm}_{\lambda_\pm}(x)$ and include all polarizations for the gauge bosons $W^\pm$.

To discuss the deviation of the production cross section $\sigma_1^{LH}(t\bar{t})$ for the process $e^+ e^- \to \nu \bar{\nu} H^0 \to \nu \bar{\nu} t \bar{t}$ in the LH model from its SM value, we define the relative correction parameter: $R_1(t\bar{t}) = \Delta \sigma_1(t\bar{t})/\sigma_1^{SM}(t\bar{t})$ with $\Delta \sigma_1(t\bar{t}) = \sigma_1^{LH}(t\bar{t}) - \sigma_1^{SM}(t\bar{t})$, in which $\sigma_1^{SM}(t\bar{t})$ denotes the production cross section for this process in the SM. Obviously, the value of the relative correction parameter $R_1(t\bar{t})$ increases as the scale parameter f decreasing. Considering the constraints from the precision measurement data on the free parameters of the LH model, we will assume $f \geq 1TeV$ in the following numerical estimation.
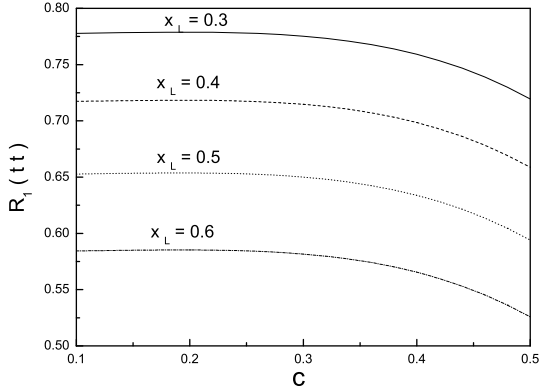

Figure 4: The parameter $R_1(t\bar{t})$ as function of c for $\nu'/\nu = \nu/5f$, $f = 1TeV$ and four values of the mixing parameter x_L.
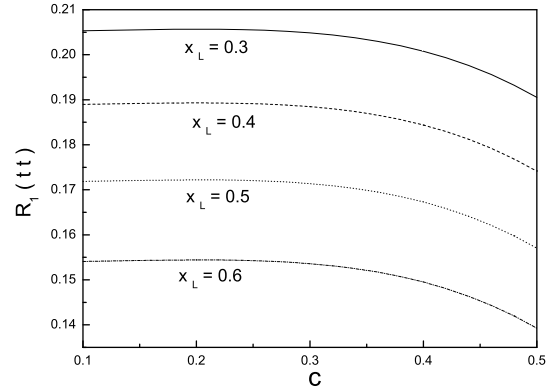
Figure 5: Same as Fig.4 but for $f = 2TeV$.

In Fig.4(Fig.5), we plot the relative correction parameter $R_1(t\bar{t})$ as a function of the mixing parameter c for $\nu'/\nu = \nu/5f$, $f = 1TeV(2TeV)$, and four values of the mixing

parameter x_L. From these figures, we can see that the relative correction parameter $R_1(t\bar{t})$ increases as the mixing parameter x_L decreasing and is insensitive to the mixing parameter c. For $f = 1TeV$, the value of $R_1(t\bar{t})$ is larger than 50% in all of the parameter space preferred by the electroweak precision data. Even if we assume the scale parameter $f = 2TeV$, the value of $R_1(t\bar{t})$ is larger than 10%.

The relative correction parameter $R_1(t\bar{t})$ is plotted in Fig.6(Fig.7) as a function of ν'/ν for $f = 1TeV(2TeV)$, $x_L = 0.5$ and three values of the mixing parameter c. From Fig.6 and Fig.7, one can see that the value of $R_1(t\bar{t})$ increases as ν'/ν increasing. As long as the scale parameter $f \leq 2TeV$, its value is larger than 10% in all of the parameter space preferred by the electroweak precision data.
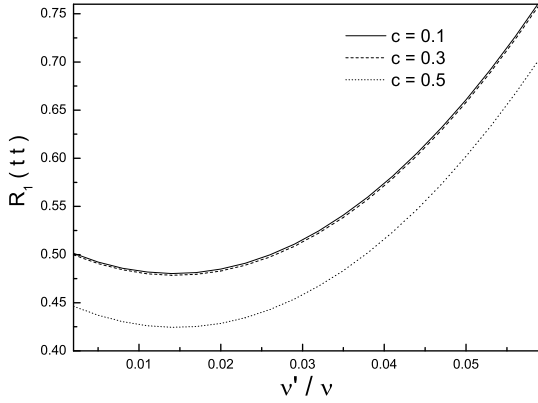

Figure 6: The parameter $R_1(t\bar{t})$ as a function of ν'/ν for $f = 1TeV$, $x_L = 0.5$ and three values of the mixing parameter c.
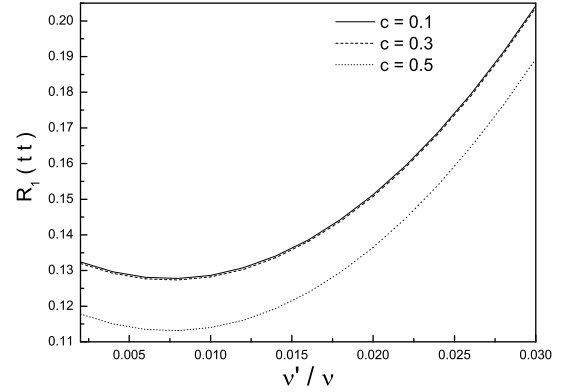
Figure 7: Same as Fig.6 but for $f = 2TeV$.

Using the EWA method[10], we have calculated the contributions of the light Higgs boson to the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$. In our numerical estimation, we have included all contributions of the longitudinal and transverse W boson components and taken the c.m. energy $\sqrt{S} = 1TeV$. However, the production cross section given by the EWA method is larger than the exact result by a factor 2 to 5, which depends on the considered c.m. energy and the Higgs boson mass[18]. Furthermore, Ref.[19] has shown that, at high energy e^+e^- colliders with c.m. energies of $1.5TeV$ or above, the effective

WW fusion calculation approximates well the exact result. Using the computer code NextCalibur[23], we have check our numerical results and find that, for $\sqrt{S} = 1TeV$ and $m_H = 115GeV$, the values of the relative correction parameter $R_1(t\bar{t})$ shown in Fig.4-Fig.7 are approximately suppressed by a factor $1/2.5$. Thus, we expect that, as long as $f \leq 1.5TeV$, the value of $R_1(t\bar{t})$ is larger than 10% in all of the parameter space preferred by the electroweak precision data.

Due to the missing momenta in the longitudinal and transverse directions, only the final $6-$jet events in which both the top and antitop decay into a b quark plus two additional quarks can be fully reconstructed experimentally[19]. The signal of the $t\bar{t}$ production via the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ should be $6-$jet events associated with large missing energy. The most dangerous backgrounds to this signal are direct $t\bar{t}$ production via the process $e^+e^- \rightarrow t\bar{t}$ and $e^+e^-t\bar{t}$ production via the process $e^+e^- \rightarrow e^+e^-t\bar{t}$. The former background can be efficiently reduced by choosing the jet association that gives the best fit to the reconstructed t and W masses and keeping events within five standard deviations of the expected values, while the latter background can be reduced by requiring the missing transverse energy in the event to be greater than $50GeV$[19]. Thus, the correction effects of the light Higgs boson on the production of the top quark pair via the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ should be detected at the future ILC experiments with $\sqrt{S} = 1TeV$.

V. The heavy gauge bosons and the process $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}t\bar{t}$

The process $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}t\bar{t}$ is one of the dominant production processes of the top quark pairs in the future ILC experiments. It is expected that there will be thousands of $t\bar{t}$ pair events produced via WW fusion process at the future ILC experiments with $\sqrt{S} = 1TeV$ and a yearly integrated luminosity $\pounds = 100fb^{-1}$. The ILC will allow the couplings of the longitudinal gauge bosons $W^{\pm}$ to the top quark to be very accurately determined[20]. Thus, this process is very sensitive to $EWSB$ mechanism and should be carefully studied within some popular specific models beyond the SM.

In the SM, the subprocess $W^+W^- \rightarrow t\bar{t}$ can proceed through the t-channel b quark exchange and the s-channel γ, Z, H^0 exchanges, which has been extensively studied in

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Refs.[20,24]. In the LH model, except for the contributions of the light Higgs boson H^0, this process receives additional contributions from the heavy photon B_H exchange and the new $SU(2)$ gauge boson Z_H exchange in the s-channel. In this section, we will consider the contributions of the gauge bosons B_H exchange and Z_H exchange to this process.

The production cross section $\sigma(S)$ of the process $e^+e^- \to \nu\bar{\nu}W^*W^* \to \nu\bar{\nu}t\bar{t}$ is dominated by collisions of two longitudinal W's at the parton level. In the following, we will first discuss the contributions of B_H exchange to this process and see whether the possible signals of B_H can be detected at the future ILC experiments with $\sqrt{S} = 1TeV$. So, as numerical estimation, we will focus our attention on the production of the top quark pairs via longitudinal gauge boson WW fusion. The main "non-standard" parts of the cross section for the subprocess $W_L^+ W_L^- \to t\bar{t}$ generated by B_H exchange can be written as:

$$
\begin{aligned}
\hat{\sigma}_{BB}(\hat{S}) &= \frac{25\pi\alpha^2}{32s_W^4}\frac{\nu^4}{f^4}(c'^2 - s'^2)^2\{[\frac{5}{6}(\frac{2}{5} - c'^2) - \frac{1}{5}x_L]^2(3 - \beta_t^2) \\
&\quad + 2[\frac{1}{5} - \frac{1}{2}c'^2 - \frac{1}{5}x_L]^2\beta_t^2\}\beta_t \cdot \frac{\hat{S}^3}{m_W^4}X_B^2,
\end{aligned}
\tag{24}
$$

$$
\hat{\sigma}_{B\gamma}(\hat{S}) = \frac{5\pi\alpha^2}{4s_W^2}\frac{\nu^2}{f^2}(c'^2 - s'^2)[\frac{5}{6}(\frac{2}{5} - c'^2) - \frac{1}{5}x_L]\beta_t(-3 + \beta_t^2) \cdot \frac{\hat{S}^2}{m_W^4}X_B,
\tag{25}
$$

$$
\begin{aligned}
\hat{\sigma}_{BZ}(\hat{S}) &= \frac{5\pi\alpha^2}{32s_W^4}\frac{\nu^2}{f^2}(c'^2 - s'^2)\{-(1 - \frac{8}{3}s_W^2)[\frac{5}{6}(\frac{2}{5} - c'^2) - \frac{1}{5}x_L](3 - \beta_t^2) \\
&\quad + 2[\frac{1}{5} - \frac{1}{2}c'^2 - \frac{1}{5}x_L]\beta_t^2\}\frac{\hat{S}^3}{m_W^4}X_B X_Z,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\hat{\sigma}_{Bb}(\hat{S}) &= \frac{15\pi\alpha^2}{16s_W^4}\frac{\nu^2}{f^2}(c'^2 - s'^2)[\frac{5}{6}(\frac{2}{5} - c'^2) - \frac{1}{5}x_L][-\frac{4}{3}\beta_t^2 - \frac{1 - \beta_t^4}{2} \\
&\quad + \frac{(1 - \beta_t^2)^3}{4\beta_t}L]\beta_t\frac{\hat{S}^2}{m_W^4} \cdot X_B
\end{aligned}
\tag{27}
$$

with

$$
X_i = \frac{\hat{S} - M_i^2}{(\hat{S} - M_i^2)^2 + M_i^2\Gamma_i^2},
\tag{28}
$$

in which Γ_i is the total decay width of the gauge boson Z or B_H. $\hat{\sigma}_{ij}(\hat{S})(i \neq j)$ denotes the interference cross section of the i and j intermediate states.
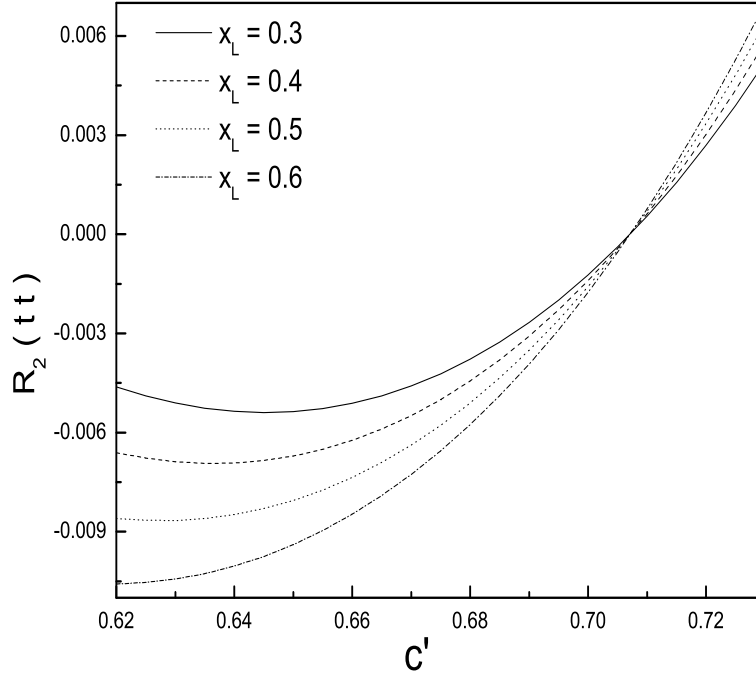
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Figure 8: The parameter $R_2(t\bar{t})$ as a function of the mixing parameter c' for $f = 1TeV$ and four values of the x_L.

We use the relative correction parameter $R_2(t\bar{t}) = \Delta\sigma_2(t\bar{t})/\sigma_2^{SM}(t\bar{t})$ to represent the contributions of B_H exchange to the process $e^+e^- \to W_L^* W_L^* \nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$, in which $\Delta\sigma_2(t\bar{t})$ denotes the corrections of B_H exchange to the SM cross section $\sigma_2^{SM}(t\bar{t})$. In Fig.8, we plot $R_2(t\bar{t})$ as a function of the mixing parameter c' for $f = 1TeV$ and four values of the mixing parameter x_L. One can see from Fig.8 that the contributions of the heavy gauge boson B_H to the process $e^+e^- \to W^* W^* \nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ depend rather significantly on the mixing parameter c'. The value of the relative correction parameter $R_2(t\bar{t})$ is positive or negative, which depends on the value of the mixing parameter c'. However, its value is very small, $|R_2(t\bar{t})| \leq 1\%$, in all of the parameter space allowed by the electroweak precision constraints. Thus, the possible signals of the gauge boson B_H can not be studied via the process $e^+e^- \to W^* W^* \nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ in the future ILC experiments.

From Eqs.(24)-(28), we can see that the contributions of the heavy photon B_H to the production cross section for the process $e^+e^- \to W^* W^* \nu\bar{\nu} \to \nu\bar{\nu}t\bar{t}$ are mainly proportional

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to the factors ν^4/f^4 and $1/(\hat{S}-M_B^2)^2$ from pure B_H contributions and to the factors ν^2/f^2 and $1/(\hat{S}-M_B^2)$ from the inference with non-B_H contributions. Furthermore, the gauge boson B_H mass M_{B_H} is proportional to f for the fixed value of c'. To see the effects of the scale parameter f on the relative correction parameter $R_2(t\bar{t})$, we plot $R_2(t\bar{t})$ as a function of f for $x_L = 0.5$ and three values of the mixing parameter c'. One can see from Fig.9 that the deviation of the production cross section from its SM value is also very small.
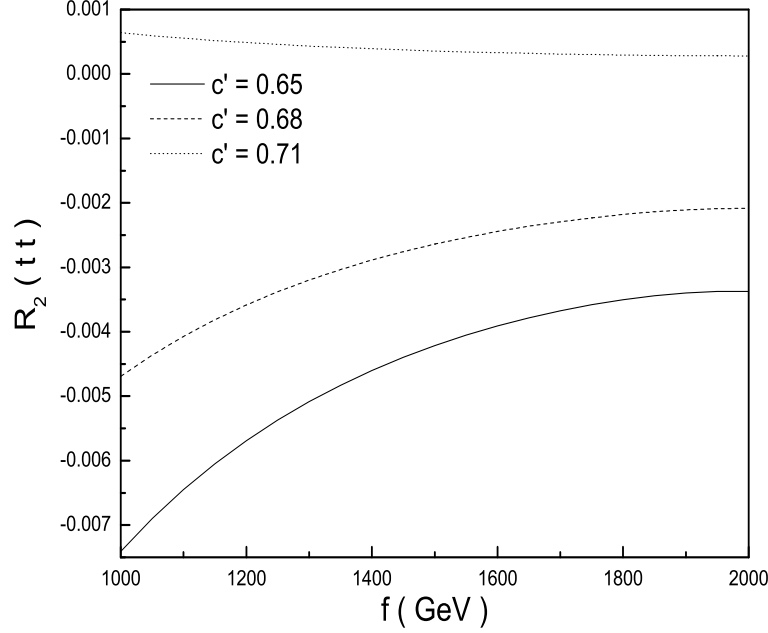

Figure 9: The parameter $R_2(t\bar{t})$ as a function of the scale parameter f for $x_L = 0.5$ and three values of the parameter c'.

Similarly to calculation for the contributions of B_H exchange to the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$, we can write the expressions of the "non-standard" parts of the production cross section for this process generated by Z_H exchange and calculate the relative deviation of the cross section from its SM value. However, our numerical results show that the contributions are also very small in all of the parameter space allowed by the electroweak precision constraints. Thus, the possible signals of the gauge bosons Z_H and B_H can not be detected via the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ in the future ILC

experiments.

VI. Conclusions and discussions

Little Higgs theory revives an old idea to keep the Higgs boson naturally light. In all of little Higgs models, there is a global symmetry structure that is broken at a scale f to make the Higgs particle as a pseudo-Goldstone boson. In general, these models predict the existence of the new heavy gauge bosons, new heavy fermions, and some of heavy triplet scalars. In this paper, we discuss the possible signals for some of these new particles predicted by the LH model via studying the WW fusion processes at the future ILC experiments with $\sqrt{S} = 1TeV$.

A future ILC will measure the production cross section of a light Higgs boson in Higgs-strahlung or WW fusion with percent-level precision[4]. In particular, WW fusion is the dominant contribution to Higgs production for $m_H < 180GeV$ at the ILC experiments with $\sqrt{S} \geq 500GeV$. We study the production of the light Higgs boson from the WW fusion process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}H^0$ in the context of the LH model and calculate the deviation of the production cross section from its SM value at the future ILC with $\sqrt{S} = 1TeV$. We find that the value of the relative correction parameter $R(H^0)$ is larger than 5% over a sizable region of the parameter space preferred by the electroweak precision data, which is comparable to the future ILC measurement precision.

In the SM, the process $W^+W^- \rightarrow t\bar{t}$ can be generated via the $t-$channel b quark exchange and $s-$channel γ, Z, H^0 exchanges. The contributions of the light Higgs boson predicted by the LH model to this process contain the pure Higgs contributions and the interference with b quark contributions. Using the EWA method, we calculate the deviation of the production cross section for the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow H^0\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ from its SM prediction. We find that the relative correction can be significantly large for reasonable values of the parameters in the LH model. For example, the value of the relative correction parameter $R_1(t\bar{t})$ is larger than 10% for $f \leq 2TeV$ in most of the parameter space, which is consistent with the electroweak precision constraints. If we use the computer code NextCalibur to give the exact cross section, then the value of $R_1(t\bar{t})$ is approximately suppressed by a factor $1/2.5$. The value of $R_1(t\bar{t})$ is larger

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than 10% for the scale parameter $f \leq 1.5 TeV$. Furthermore, the main backgrounds, $t\bar{t}$ and $e^+e^-t\bar{t}$ production, to the signal of the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$ can be efficiently reduced by the suitably cuts. Thus, the correction effects can be seen as new signals of the light Higgs boson and should be detected via this process at the future ILC experiments with $\sqrt{S} = 1 TeV$.

The heavy gauge bosons Z_H and B_H can produce the corrections to the process $e^+e^- \rightarrow W^*W^*\nu\bar{\nu} \rightarrow \nu\bar{\nu}t\bar{t}$. However, our numerical results show that the correction effects are very small in all of the parameter space preferred by the electroweak precision data. Thus, these new particles can not produce observable signals via this process at the future ILC experiments.

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